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Nearly degenerate mass and bimaximal mixing of neutrinos in the SO(3) gauge model of leptons

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Abstract. A realistic scheme for masses and mixing of leptons is investigated in the model with gauged SO(3) lepton flavor symmetry. Within this scheme, a nearly bimaximal neutrino mixing pattern with a maximal CP-violating phase is found to be the only possible solution for reconciling both solar and atmospheric neutrino flux anomalies. Three Majorana neutrino masses are nearly degenerate and large enough to play a significant cosmological role. Lepton flavor-violating effects via SO(3) gauge interactions can be as large as the present experimental limits. Masses of the SO(3) gauge bosons are bounded to be above 24 TeV when the SO(3) gauge boson mixing angle θ_F and coupling constant g_3' are taken to be the same as those (θ_W and g) in the electroweak theory.

Evidence for oscillation of atmospheric neutrinos (and hence nonzero neutrino mass) reported recently by the Superkamiokande Collaboration [1] is thought as a major milestone in the search for new physics beyond the standard model(SM). Massive neutrinos are also regarded as the best candidate for hot dark matter, and may play an essential role in the evolution of the large-scale structure of the universe [3]. The nonzero neutrino mass can also provide a natural explanation for the solar neutrino problem which is in fact the first indication of neutrino oscillation [4]. The solar neutrino flux measured recently by the Superkamiokande Collaboration [2] is only about 37% of that expected from the standard solar model [5]. The SM has been tested by more and more precise experiments; its greatest success is the gauge symmetry structure $SU_{\rm L}(2)\times U_Y(1)$. Nevertheless, neutrinos are assumed to be massless in the SM. To introduce masses and mixings of the neutrinos, it is necessary to modify and go beyond the SM. As a simple extension of the SM, it is of interest to investigate possible flavor symmetries among three families of leptons. In a recent paper [6], we have introduced gauged SO(3) symmetry for the three lepton families and observed that it has some remarkable features which are applicable to the current interesting phenomena concerning neutrinos. As the first essential step, it has been shown [6] that the SO(3) gauge symmetry allows three Majorana neutrino masses to be nearly degenerate[1] and large enough for hot dark matter. The nearly bimaximal mixing patterns (which include the bimaximal mixing pattern [11,12] and the democratic mixing pattern [13,14]) with maximal CP-violating phases reconcile both solar and atmospheric neutrino flux anomalies. As the vacuum structure of spontaneous SO(3) gauge symmetry breaking can automatically generate a maximal CP-violating phase, the scheme can be made to be consistent with the neutrinoless double beta decay; this leads to the Georgi–Glashow form for the neutrino mass matrix [15]. In this paper, we are going to further investigate such a gauge model and show how to carry out other necessary steps to realize a realistic scheme for masses and mixing of the leptons. As was expected in [6], the realistic scheme does not significantly change the main interesting features mentioned there. In particular, it will be seen that within the realistic scheme presented here, the bimaximal mixing pattern becomes the only possible solution for reconciling both solar and atmospheric neutrino data.

For a more simple and model-independent consideration, we shall start directly from the following $\mathrm{SO(3)}_F\times \mathrm{SU(2)_L}\times\mathrm{U(1)}_Y$ invariant effective Lagrangian for leptons,

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}g_3' A_\mu^k \left(\bar{L}_i\gamma^\mu (t^k)_{ij}L_j + \bar{e}_{Ri}\gamma^\mu (t^k)_{ij}e_{Rj}\right) \\ & + D_\mu\varphi^* D^\mu\varphi + D_\mu\varphi^{'*}D^\mu\varphi' \\ & + \left(C_1\frac{\varphi_i\varphi_j}{M_1M_2} + C_1'\frac{\varphi_i'\varphi_j'}{M_1'M_2'}\frac{\chi}{M} + C_1''\frac{\chi'}{M'}\delta_{ij}\right) \\ & \times\bar{L}_i\phi_1 e_{R\ j} + \text{H.c.} \\ & + \left(C_0\delta_{ij} + C_0'\frac{\varphi_i\varphi_j^*}{M_2^2} + C_0''\frac{\varphi_i'\varphi_j^{'*}}{M_2^{'2}}\right) \\ & \times\frac{1}{M_N}\bar{L}_i\phi_2\phi_2^T L_j^c + \text{H.c.} + \mathcal{L}_{\rm SM}, \qquad (1)\end{aligned}$$

which is assumed to result from the integration out of heavy particles. $\mathcal{L}_{\rm SM}$ denotes the Lagrangian of the standard model; $\bar{L}_i(x) = (\bar{\nu}_i,\bar{e}_i)_{\rm L}$ (i=1,2,3) are the $\mathrm{SU(2)_L}$ doublet leptons; $e_{R\ i}$ (i=1,2,3) are the three right-handed

[1] Recently, authors in [7–10] have also discussed SO(3) flavor symmetry in connection with nearly degenerate neutrinos.

charged leptons; $\phi_1(x)$ and $\phi_2(x)$ are two Higgs doublets; $\varphi^T = (\varphi_1(x), \varphi_2(x), \varphi_3(x))$ and $\varphi'^T = (\varphi'_1(x), \varphi'_2(x), \varphi'_3(x))$ are two complex SO(3) triplet scalars; $\chi(x)$ and $\chi'(x)$ are two singlet scalars; M_1, M_2, M, M'_1, M'_2, M'; and M_N are possible mass scales concerning heavy fermions; and C_a, C'_a, and C''_a $(a = 0, 1)$ are six coupling constants. One can obtain the structure of the above effective Lagrangian by imposing an additional U(1) symmetry; this is analogous to the construction of the C_0 and C_1 terms discussed in detail in [6]. After the symmetry $\mathrm{SO}(3)_F\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_Y$ is broken down to the $\mathrm{U}(1)_{em}$ symmetry, we obtain mass matrices of the neutrinos and charged leptons as follows:

$$
\begin{aligned}
(M_e)_{ij} &= m_1 \frac{\hat{\sigma}_i\hat{\sigma}_j}{\sigma^2} + m'_1 \frac{\hat{\sigma}'_i\hat{\sigma}'_j}{\sigma'^2} + m''_1 \delta_{ij}, \\
(M_\nu)_{ij} &= m_0\delta_{ij} + m'_0 \frac{\hat{\sigma}_i\hat{\sigma}^*_j + \hat{\sigma}_j\hat{\sigma}^*_i}{2\sigma^2} \\
&\quad + m''_0 \frac{\hat{\sigma}'_i\hat{\sigma}'^*_j + \hat{\sigma}'_j\hat{\sigma}'^*_i}{2\sigma'^2},
\end{aligned} \tag{2}
$$

where the mass matrices M_e and M_ν are defined in the basis $\mathcal{L}_M = \bar{e}_\mathrm{L} M_e e_\mathrm{R} + \bar{\nu}_\mathrm{L} M_\nu \nu^c_\mathrm{L} + \mathrm{H.c.}$. The constants $\hat{\sigma}_i = \langle\varphi_i(x)\rangle$ and $\hat{\sigma}'_i = \langle\varphi'_i(x)\rangle$ represent the vacuum expectation values of the two triplet scalars $\varphi(x)$ and $\varphi'(x)$. The six mass parameters are defined as: $m_0 = C_0 v_2^2/M_N$, $m'_0 = C'_0(\sigma^2/M_2^2)(v_2^2/M_N)$, $m''_0 = C''_0(\sigma'^2/M_2'^2)(v_2^2/M_N)$, $m_1 = C_1 v_1\sigma^2/M_1M_2$, $m'_1 = C'_1(\xi/M)(v_1\sigma^2/M'_1M'_2)$ and $m''_1 = C''_1 v_1\xi'/M$. Here $\sigma = \sqrt{|\hat{\sigma}_1|^2 + |\hat{\sigma}_2|^2 + |\hat{\sigma}_3|^2}$, and $\sigma' = \sqrt{|\hat{\sigma}'_1|^2 + |\hat{\sigma}'_2|^2 + |\hat{\sigma}'_3|^2}$. $\xi = \langle\chi(x)\rangle$ and $\xi' = \langle\chi'(x)\rangle$ denote the vacuum expectation values of the two singlet scalars.

Utilizing the gauge symmetry property, it is convenient to reexpress the complex triplet scalar fields $\varphi_i(x)$ and $\varphi'_i(x)$ in terms of the SO(3) rotational fields $O(x) = \mathrm{e}^{\mathrm{i}\eta_i(x)t^i}$, $O'(x) = \mathrm{e}^{\mathrm{i}\eta'_i(x)t^i} \in \mathrm{SO}(3)$:

$$
\begin{aligned}
\begin{pmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{pmatrix} &= \mathrm{e}^{\mathrm{i}\eta_i(x)t^i} \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1(x) \\ \mathrm{i}\rho_2(x) \\ \rho_3(x) \end{pmatrix} \\
\begin{pmatrix} \varphi'_1(x) \\ \varphi'_2(x) \\ \varphi'_3(x) \end{pmatrix} &= \mathrm{e}^{\mathrm{i}\eta'_i(x)t^i} \frac{1}{\sqrt{2}} \begin{pmatrix} \rho'_1(x) \\ \mathrm{i}\rho'_2(x) \\ \rho'_3(x) \end{pmatrix}
\end{aligned} \tag{3}
$$

where the three rotational fields $\eta_i(x)$ $(\eta'_i(x))$ and the three amplitude fields $\rho_i(x)$ $(\rho'_i(x))$ reparameterize the six real fields of the complex triplet scalar field $\varphi(x)$ $(\varphi(x))$. Here, the imaginary part is assigned to the second amplitude field[2]. SO(3) gauge symmetry allows one to remove three degrees of freedom from the six rotational fields. Thus the vacuum structure of the SO(3) symmetry is determined by only nine degrees of freedom. These nine degrees of freedom can be taken as $\rho_i(x)$, $\rho'_i(x)$, and $(\eta_i(x) - \eta'_i(x))$ without loss of generality. Here we will consider the following vacuum structure for the SO(3) symmetry breaking:

$$
\begin{aligned}
&\langle\rho_i(x)\rangle = \sigma_i, \qquad \langle\rho'_i(x)\rangle = \sigma'_i, \\
&\langle(\eta_i(x) - \eta'_i(x))\rangle = 0.
\end{aligned} \tag{4}
$$

With this vacuum structure, the mass matrices of the neutrinos and charged leptons can be reexpressed as

$$
\begin{aligned}
M_e &= m_1 \begin{pmatrix} s_1^2 s_2^2 & \mathrm{i}c_1 s_1 s_2^2 & s_1 c_2 s_2 \\ \mathrm{i}c_1 s_1 s_2^2 & -c_1^2 s_2^2 & \mathrm{i}c_1 c_2 s_2 \\ s_1 c_2 s_2 & \mathrm{i}c_1 c_2 s_2 & c_2^2 \end{pmatrix} \\
&+ m'_1 \begin{pmatrix} s_1'^2 s_2'^2 & \mathrm{i}c'_1 s'_1 s_2'^2 & s'_1 c'_2 s'_2 \\ \mathrm{i}c'_1 s'_1 s_2'^2 & -c_1'^2 s_2'^2 & \mathrm{i}c'_1 c'_2 s'_2 \\ s'_1 c'_2 s'_2 & \mathrm{i}c'_1 c'_2 s'_2 & c_2'^2 \end{pmatrix} \\
&+ m''_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned} \tag{5}
$$

and

$$
\begin{aligned}
M_\nu &= m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + m'_0 \begin{pmatrix} s_1^2 s_2^2 & 0 & s_1 c_2 s_2 \\ 0 & c_1^2 s_2^2 & 0 \\ s_1 c_2 s_2 & 0 & c_2^2 \end{pmatrix} \\
&+ m''_0 \begin{pmatrix} s_1'^2 s_2'^2 & 0 & s'_1 c'_2 s'_2 \\ 0 & c_1'^2 s_2'^2 & 0 \\ s'_1 c'_2 s'_2 & 0 & c_2'^2 \end{pmatrix}
\end{aligned} \tag{6}
$$

where $s_1 = \sin\theta_1 = \sigma_1/\sigma_{12}$ and $s_2 = \sin\theta_2 = \sigma_{12}/\sigma$ with $\sigma_{12} = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\sigma = \sqrt{\sigma_{12}^2 + \sigma_3^2}$. We define s'_1 and s'_2 similarly.

Note that the two nondiagonal matrices in the mass matrix M_e are rank 1 matrices. It is interesting to observe that when the four angles θ_1, θ_2, θ'_1 and θ'_2 satisfy the conditions

$$
\frac{s_1}{c_1} = \frac{s'_1}{c'_1}, \qquad \frac{c_2}{s_2} = -\frac{s'_2}{c'_2}, \tag{7}
$$

which are equivalent to $\sigma'_1/\sigma'_2 = \sigma_1/\sigma_2$, $\sigma'_{12}/\sigma'_3 = -\sigma_3/\sigma_{12}$, the two nondiagonal matrices in the mass matrix M_e can be simultaneously diagonalized by a unitary matrix U_e via $M'_e = U_e^\dagger M_e U_e^*$. Here,

$$
M'_e = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m'_1 & 0 \\ 0 & 0 & m_1 \end{pmatrix} + m''_1 U_e^\dagger U_e^* \tag{8}
$$

and

$$
U_e^\dagger = \begin{pmatrix} \mathrm{i}c_1 & -s_1 & 0 \\ c_2 s_1 & -\mathrm{i}c_1 c_2 & -s_2 \\ s_1 s_2 & -\mathrm{i}c_1 s_2 & c_2 \end{pmatrix} \tag{9}
$$

where $U_e^\dagger U_e^*$ has the following explicit form:

$$
\begin{aligned}
&U_e^\dagger U_e^* \\
&= \begin{pmatrix} s_1^2 - c_1^2 & 2\mathrm{i}c_1 s_1 c_2 & 2\mathrm{i}c_1 s_1 s_2 \\ 2\mathrm{i}c_1 s_1 c_2 & c_2^2(s_1^2 - c_1^2) + s_2^2 & c_2 s_2(s_1^2 - c_1^2) - c_2 s_2 \\ 2\mathrm{i}c_1 s_1 s_2 & c_2 s_2(s_1^2 - c_1^2) - c_2 s_2 & s_2^2(s_1^2 - c_1^2) + c_2^2 \end{pmatrix}.
\end{aligned} \tag{10}
$$

[2] The other two possible assignments and corresponding consequences will be discussed elsewhere [16]

The hierarchical structure of the charged lepton mass implies that $m_1'' << m_1' << m_1$; it is then not difficult to see that the matrix M_e' will be further diagonalized by a unitary matrix U_e' via $D_e = U_e'^{\dagger} M_e' U_e'^{*} = U_e'^{\dagger} U_e^{\dagger} M_e U_e^{*} U_e'^{*}$ with

$$D_e = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \tag{11}$$

and

$$U_e'^{\dagger} = \begin{pmatrix} 1+O(m_1''/m_1') & \mathrm{i}O(m_1''/m_1') & \mathrm{i}O(m_1''/m_1) \\ \mathrm{i}O(m_1''/m_1') & 1+O(m_1''/m_1') & O(m_1''/m_1) \\ \mathrm{i}O(m_1''/m_1) & O(m_1''/m_1) & 1+O(m_1''/m_1), \end{pmatrix} \tag{12}$$

where $m_e = O(m_1'')$, $m_\mu = m_1' + O(m_1'')$ and $m_\tau = m_1 + O(m_1'')$ define the three charged lepton masses. This indicates that the unitary matrix U_e' does not significantly differ from the unit matrix. Applying the same conditions given in (7), the neutrino mass matrix can be rewritten as

$$M_\nu = m_0 \begin{pmatrix} 1+\Delta_- s_1^2 & 0 & 2\delta_- s_2 c_2 s_1 \\ 0 & 1+\Delta_- c_1^2 & 0 \\ 2\delta_- s_2 c_2 s_1 & 0 & 1+\Delta_+ \end{pmatrix}, \tag{13}$$

with

$$\Delta_\pm = \delta_+ \pm \delta_- \cos 2\theta_2, \qquad \delta_\pm = (m_0' \pm m_0'')/2m_0. \tag{14}$$

This neutrino mass matrix can be easily diagonalized by an orthogonal matrix O_ν via $O_\nu^{\mathrm{T}} M_\nu O_\nu$. Explicitly, the matrix O_ν is found to be

$$O_\nu = \begin{pmatrix} c_\nu & 0 & s_\nu \\ 0 & 1 & 0 \\ -s_\nu & 0 & c_\nu \end{pmatrix}, \tag{15}$$

with

$$\tan 2\theta_\nu = 2\delta_- s_1 \sin 2\theta_2/(\Delta_+ - \Delta_- s_1^2). \tag{16}$$

For the physical mass basis of the neutrinos and charged leptons, we then obtain the CKM-type lepton mixing matrix U_{LEP} appearing in the interactions of the charged weak gauge bosons and leptons, i.e., $\mathcal{L}_W = \bar{e}_{\mathrm{L}} \gamma^\mu U_{\mathrm{LEP}} \nu_{\mathrm{L}} W_\mu^- + \mathrm{H.c.}$ Explicitly, we have

$$U_{\mathrm{LEP}} = U_e'^{\dagger} U_e^{\dagger} O_\nu = U_e'^{\dagger} \begin{pmatrix} \mathrm{i}c_1 c_\nu & -s_1 & \mathrm{i}c_1 s_\nu \\ c_2 s_1 c_\nu + s_2 s_\nu & -\mathrm{i}c_1 c_2 & c_2 s_1 s_\nu - s_2 c_\nu \\ s_1 s_2 c_\nu - c_2 s_\nu & -\mathrm{i}c_1 s_2 & s_1 s_2 s_\nu + c_2 c_\nu \end{pmatrix} \tag{17}$$

The three neutrino masses are found to be

$$\begin{aligned} m_{\nu_e} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- s_1^2) \\ &\quad - \frac{1}{2}(\Delta_+ - \Delta_- s_1^2)\sqrt{1+\tan^2 2\theta_\nu}\ ] \\ m_{\nu_\mu} &= m_0[1 + \Delta_- c_1^2] \\ m_{\nu_\tau} &= m_0[1 + \frac{1}{2}(\Delta_+ + \Delta_- s_1^2) \\ &\quad + \frac{1}{2}(\Delta_+ - \Delta_- s_1^2)\sqrt{1+\tan^2 2\theta_\nu}\ ]. \end{aligned} \tag{18}$$

For $\tan^2 2\theta_\nu << 1$, masses of the three neutrinos are simply given by

$$\begin{aligned} m_{\nu_e} &\simeq m_0[1 + \Delta_- s_1^2 - \frac{1}{4}(\Delta_+ - \Delta_- s_1^2)\tan^2 2\theta_\nu\ ] \\ m_{\nu_\mu} &\simeq m_0[1 + \Delta_- c_1^2], \\ m_{\nu_\tau} &\simeq m_0[1 + \Delta_+ + \frac{1}{4}(\Delta_+ - \Delta_- s_1^2)\tan^2 2\theta_\nu\ ] \end{aligned} \tag{19}$$

from which one easily reads off the mass-squared differences

$$\begin{aligned} \Delta m_{\mu e}^2 &= m_{\nu_\mu}^2 - m_{\nu_e}^2 \simeq m_0^2[\Delta_-(c_1^2 - s_1^2) \\ &\quad + \frac{1}{4}(\Delta_+ - \Delta_- s_1^2)\tan^2 2\theta_\nu][2 + \Delta_-] \\ \Delta m_{\tau\mu}^2 &= m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \simeq m_0^2[\Delta_+ - \Delta_- c_1^2 \\ &\quad + \frac{1}{4}(\Delta_+ - \Delta_- s_1^2)\tan^2 2\theta_\nu][2 + \Delta_+ + \Delta_- c_1^2] \end{aligned} \tag{20}$$

It is noticed that when $\sin\theta_\nu << 1$, we are led to a nearly two-flavor mixing scheme. From the recent atmospheric neutrino data [1], which suggested a large neutrino mixing between ν_μ and ν_τ, i.e., the relevant mixing angle satisfies $\sin^2 2\theta > 0.8$, we then obtain almost the same constraint on θ_2 when neglecting other small mixing angles:

$$\sin^2 2\theta_2 > 0.8. \tag{21}$$

Thus the condition $\sin\theta_\nu << 1$ is equivalent to $\delta_- s_1 \ll \delta_+ c_1^2$, and we have, to a good approximation, the simple relations: $\Delta_+ \simeq \Delta_- \simeq \delta_+$ and $\tan 2\theta_\nu \simeq 2\delta_- s_1/\delta_+ c_1^2$. With this approximation, the neutrino mass-squared differences become more simple

$$\begin{aligned} \Delta m_{\mu e}^2 &= m_{\nu_\mu}^2 - m_{\nu_e}^2 \\ &\simeq m_0^2 \delta_+[c_1^2 - s_1^2 + (\delta_- s_1)^2/(\delta_+ c_1)^2][2 + \delta_+] \\ \Delta m_{\tau\mu}^2 &= m_{\nu_\tau}^2 - m_{\nu_\mu}^2 \\ &\simeq m_0^2 \delta_+[s_1^2 + (\delta_- s_1)^2/(\delta_+ c_1)^2][2 + \delta_+(1 + c_1^2)] \end{aligned} \tag{22}$$

It has been shown [1,17,18] that for the atmospheric neutrino anomaly to be explained, the required neutrino mass-squared difference $\Delta m_{\tau\mu}^2$ favors the range

$$5 \times 10^{-4}\mathrm{eV}^2 < \Delta m_{\tau\mu}^2 < 6 \times 10^{-3}.\mathrm{eV}^2 \tag{23}$$

For the observed deficit of the solar neutrino fluxes in comparison with the solar neutrino fluxes computed from the solar standard model to be understood [5], the neutrino mass- squared difference $\Delta m_{\mu e}^2$ is required to fall into the range [17]:

$$6 \times 10^{-11}\mathrm{eV}^2 < \Delta m_{\mu e}^2 < 2 \times 10^{-5}\mathrm{eV}^2. \tag{24}$$

Here, the larger and smaller values of $\Delta m^2_{\mu e}$ provide Mikheyev–Smirnov–Wolfenstein [19] and just-so [20] explanations, respectively, for the solar neutrino puzzle. It is seen that the ratio between the two mass-squared differences must satisfy $\Delta m^2_{\mu e}/\Delta m^2_{\tau\mu} < 0.04$. With this condition and $\delta_- << \delta_+$, we then obtain from (22) the following constraint on the mixing angle θ_1:

$$|c_1^2/s_1^2 - 1| < 0.04 \qquad (25)$$

Note that this constraint is independent of the mass scale m_0. With these constraints, we arrive at the following relations:

$$\frac{m_1''}{m_1'} \sim \sqrt{\frac{m_e}{m_\mu}} = 0.07, \qquad \frac{m_1''}{m_1} \sim \frac{\sqrt{m_e m_\mu}}{m_\tau} = 0.004. \quad (26)$$

Due to the smallness of the mixing angles in U_e' and θ_ν, we may conclude that the neutrino mixing between ν_e and ν_μ is almost maximal:

$$\sin^2 2\theta_1 > 0.998. \qquad (27)$$

This may leave just-so oscillations as the only viable explanation of the solar neutrino data, as can be seen from the analyses in [21]. This requires that $\sigma_1 \simeq \sigma_2$ and $m_0' \simeq m_0''$; these may need fine-tuning if they are not ensured by symmetries.

With the above analyses, we may come to the conclusion that with two-flavor mixing and the hierarchical mass-squared differences $\Delta m^2_{\mu e} << \Delta m^2_{\tau\mu}$, the present scheme favors a bimaximal neutrino mixing pattern for the explanations of the solar and atmospheric neutrino flux anomalies.

It is not difficult to show that the resulting bimaximal neutrino mixing pattern allows the three neutrino masses to be nearly degenerate and large enough for hot dark matter without conflict with the current data on neutrinoless double beta decay. This can be seen from the fact that failure to detect neutrinoless double beta decay leads to bounds on an effective electron neutrino mass $\langle m_{\nu_e}\rangle = \sum_i m_{\nu_i}(U_{\mathrm{LEP}})^2_{ei} < 0.46$ eV [22]. To a good approximation, when neglecting the small mixing angles in U_e', we obtain

$$\langle m_{\nu_e}\rangle \simeq m_0|s_1^2 - c_1^2| < 0.46\mathrm{eV} \qquad (28)$$

Assuming that neutrino masses are large enough to play an essential role in the evolution of the large-scale structure of the universe, we may set $m_0 \sim 2$ eV, and thus the above constraint will result in the a bound on the mixing angle θ_1,

$$|s_1^2 - c_1^2| < 0.23, \qquad (29)$$

which is weaker than the one given in (25).

The smallness of the mass-squared difference $\Delta m^2_{\mu e}$ implies that $\sin\theta_\nu < 0.001$ for $m_0 \sim 2$ eV. To a good approximation, we may neglect the small mixing angle θ_ν and the small mixing of order m_1''/m_1 in U_e'. With these considerations, the CKM-type lepton mixing matrix is simply given by

$$U_{\mathrm{LEP}} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\mathrm{i} & -\frac{1}{\sqrt{2}} & -\mathrm{i}\sqrt{\frac{m_e}{m_\mu}}s_2 \\ \frac{1}{\sqrt{2}}c_2 & -\frac{1}{\sqrt{2}}c_2\mathrm{i} & -s_2 \\ \frac{1}{\sqrt{2}}s_2 & -\frac{1}{\sqrt{2}}s_2\mathrm{i} & c_2 \end{pmatrix}, \qquad (30)$$

which arrives at the pattern suggested in [23] when the small mixing angle at the order of magnitude $\sqrt{m_e/m_\mu}$ is neglected. Going back to the weak gauge and charged-lepton mass basis, we find that the neutrino mass matrix has the following simple form:

$$M_\nu \simeq m_0 \begin{pmatrix} -\frac{m_e}{m_\mu}s_2^2 & \mathrm{i}c_2 & \mathrm{i}s_2 \\ \mathrm{i}c_2 & s_2^2 & -c_2 s_2 \\ \mathrm{i}s_2 & -c_2 s_2 & c_2^2 \end{pmatrix}. \qquad (31)$$

Going on the recent atmospheric neutrino data, we are motivated to consider two particular interesting cases: First, setting the vacuum expectation values to be $\sigma_3^2 = \sigma_1^2 + \sigma_2^2$ and $\sigma_1 = \sigma_2$, namely, $s_1 = s_2 = 1/\sqrt{2}$ ($\sin^2 2\theta_1 = \sin^2 2\theta_2 = 1$), we then obtain a realistic bimaximal mixing pattern with a maximal CP-violating phase. Explicitly, the neutrino mass and mixing matrices read

$$M_\nu \simeq m_0 \begin{pmatrix} -0.002 & \frac{1}{\sqrt{2}}\mathrm{i} & \frac{1}{\sqrt{2}}\mathrm{i} \\ \frac{1}{\sqrt{2}}\mathrm{i} & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}}\mathrm{i} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad (32)$$

and

$$U_{\mathrm{LEP}} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\mathrm{i} & -\frac{1}{\sqrt{2}} & -0.05\mathrm{i} \\ \frac{1}{2} & -\frac{1}{2}\mathrm{i} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2}\mathrm{i} & \frac{1}{\sqrt{2}} \end{pmatrix}; \qquad (33)$$

When neglecting the small mixing angle at the order of magnitude $\sqrt{m_e/m_\mu}$, we derive the pattern suggested by Georgi and Glashow [15].

Second, setting the three vacuum expectation values σ_i (i=1,2,3) to be democratic, i.e., $\sigma_1 = \sigma_2 = \sigma_3$, hence $s_1 = 1/\sqrt{2}$ and $s_2 = \sqrt{2/3}$ ($\sin^2 2\theta_1 = 1$ and $\sin^2 2\theta_2 = 0.89$), we then arrive at a realistic democratic mixing pattern with a maximal CP-violating phase. The explicit neutrino mass and mixing matrices are given by

$$M_\nu \simeq m_0 \begin{pmatrix} -0.003 & \frac{1}{\sqrt{3}}\mathrm{i} & \frac{2}{\sqrt{6}}\mathrm{i} \\ \frac{1}{\sqrt{3}}\mathrm{i} & \frac{2}{3} & -\frac{\sqrt{2}}{3} \\ \frac{2}{\sqrt{6}}\mathrm{i} & -\frac{\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix} \qquad (34)$$

and

$$U_{\mathrm{LEP}} \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\mathrm{i} & -\frac{1}{\sqrt{2}} & -0.057\mathrm{i} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}}\mathrm{i} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}\mathrm{i} & \frac{1}{\sqrt{3}} \end{pmatrix}; \qquad (35)$$

when further neglecting the small mixing angle at the order of magnitude $\sqrt{m_e/m_\mu}$, we obtain a similar form to that provided by Mohapatra [24].

$$M_F^2 = m_F^2 \begin{pmatrix} 1+\xi & 0 & -s_1(\frac{c_2}{s_2}-\frac{s_2}{c_2}\xi) \\ 0 & (s_1^2+\frac{c_2^2}{s_2^2})+(c_1^2+\frac{s_2^2}{c_2^2})\xi & 0 \\ -s_1(\frac{c_2}{s_2}-\frac{s_2}{c_2}\xi) & 0 & (c_1^2+\frac{c_2^2}{s_2^2})+(s_1^2+\frac{s_2^2}{c_2^2})\xi \end{pmatrix} \quad (41)$$

From the hierarchical feature in Δm^2, i.e., $\Delta m^2_{\mu e} << \Delta m^2_{\tau\mu} \simeq \Delta m^2_{\tau e}$, and the nearly bimaximal mixing pattern, formulas for the oscillation probabilities can be greatly simplified to

$$P_{\nu_e\to\nu_e}|_{\mathrm{solar}} \simeq 1-4|U_{e1}|^2|U_{e2}|^2\sin^2\left(\frac{\Delta m^2_{\mu e}L}{4E}\right)$$
$$P_{\nu_\mu\to\nu_\mu}|_{\mathrm{atmospheric}} \simeq 1-4(1-|U_{\mu 3}|^2)|U_{\mu 3}|^2 \times\sin^2\left(\frac{\Delta m^2_{\tau\mu}L}{4E}\right),$$
$$P_{\nu_\beta\to\nu_\alpha} \simeq 4|U_{\beta 3}|^2|U_{\alpha 3}|^2\sin^2\left(\frac{\Delta m^2_{\tau\mu}L}{4E}\right), \quad (36)$$

and

$$\frac{P_{\nu_\mu\to\nu_e}}{P_{\nu_\mu\to\nu_\tau}}|_{\mathrm{atmospheric}} \simeq \frac{|U_{e3}|^2}{|U_{\tau 3}|^2} << 1. \quad (37)$$

This may present the simplest scheme for reconciling both solar and atmospheric neutrino fluxes via oscillations of three neutrinos.

On the other hand, the three nearly degenerate neutrino masses can be large enough for hot dark matter. The relation between the total neutrino mass $m(\nu)$ and the fraction Ω_ν of critical density that neutrinos contribute is [3]

$$\frac{\Omega_\nu}{\Omega_m} = 0.03\frac{m(\nu)}{1\ \mathrm{eV}}\left(\frac{0.6}{h}\right)^2\frac{1}{\Omega_m} \simeq 0.09\frac{m_0}{1\ \mathrm{eV}}\left(\frac{0.6}{h}\right)^2\frac{1}{\Omega_m} \quad (38)$$

with $h = 0.5-0.8$ the expansion rate of the universe (Hubble constant H_0) in units of 100 km/s/Mpc. Ω_m is the fraction of critical density that matter contributes. For $m_0 \sim 2$ eV and $h = 0.6$, the fraction $\Omega_\nu \simeq 18\%$ for $\Omega_m = 1$.

We now come to discuss SO(3) gauge interactions in the present scheme. Explicitly, the SO(3) gauge interactions in the mass eigenstate of the leptons have the following form:

$$\mathcal{L}_F = \frac{1}{2}g_3' A^i_\mu\left(\bar\nu_{\mathrm{L}} t^i\gamma^\mu\nu_{\mathrm{L}} + \bar e_{\mathrm{L}} K^i_e\gamma^\mu e_{\mathrm{L}} - \bar e_{\mathrm{R}} K^{i*}_e\gamma^\mu e_{\mathrm{R}}\right) \quad (39)$$

with $K^i_e = U_e^{'\dagger} U_e^\dagger t^i U_e U'_e$. After the SO(3) gauge symmetry is spontaneously broken down, the gauge fields A^i_μ receive masses by "eating" three of the rotational fields. For the SO(3) vacuum structure given above, A^1_μ and A^3_μ are not in the mass eigenstates since they mix with each other. The mass matrix of the gauge fields A^i_μ is found to be

$$M_F^2 = \frac{1}{4}g_3^{'2}\begin{pmatrix} \sigma_{12}^2+\sigma_{12}^{'2} & 0 & -(\sigma_1\sigma_3+\sigma'_1\sigma'_3) \\ 0 & \sigma_{13}^2+\sigma_{13}^{'2} & 0 \\ -(\sigma_1\sigma_3+\sigma'_1\sigma'_3) & 0 & \sigma_{23}^2+\sigma_{23}^{'2} \end{pmatrix} \quad (40)$$

with $\sigma_{ij}^2 = \sigma_i^2+\sigma_j^2$. By using the conditions given in (7), the above mass matrix reads (see (41) on top of the page) with $m_F^2 = g_3^{'2}\sigma_{12}^2/4$ and $\xi = \sigma_{12}^{'2}/\sigma_{12}^2$. This mass matrix is diagonalized by an orthogonal matrix O_F via $O_F^T M_F^2 O_F$. Denoting the physical gauge fields as F^{i}_μ, we then have $A^{\mathrm{i}}_\mu = O_F^{\mathrm{i}j}F^j_\mu$. Explicitly,

$$\begin{pmatrix} A^1_\mu \\ A^2_\mu \\ A^3_\mu \end{pmatrix} = \begin{pmatrix} c_F & 0 & -s_F \\ 0 & 1 & 0 \\ s_F & 0 & c_F \end{pmatrix}\begin{pmatrix} F^1_\mu \\ F^2_\mu \\ F^3_\mu \end{pmatrix}, \quad (42)$$

with $c_F \equiv \cos\theta_F$ and $s_F \equiv \sin\theta_F$. The mixing angle θ_F is given by

$$\tan 2\theta_F = \frac{2s_1(\frac{c_2}{s_2}-\frac{s_2}{c_2}\xi)}{(\frac{c_2^2}{s_2^2}-s_1^2)+(\frac{s_2^2}{c_2^2}-c_1^2)\xi}. \quad (43)$$

Masses of the three physical gauge bosons F^{i}_μ are found to be

$$\begin{aligned} m^2_{F_1} &= \frac{m_F^2}{2}\left(\left(c_1^2+\frac{1}{s_2^2}\right)+\left(s_1^2+\frac{1}{c_2^2}\right)\xi - \left[\left(\frac{c_2^2}{s_2^2}-s_1^2\right)+\left(\frac{s_2^2}{c_2^2}-c_1^2\right)\xi\right]\sqrt{1+\tan^2 2\theta_F}\right) \\ m^2_{F_2} &= m_F^2\left(\left(s_1^2+\frac{c_2^2}{s_2^2}\right)+\left(c_1^2+\frac{s_2^2}{c_2^2}\right)\xi\right) \\ m^2_{F_3} &= \frac{m_F^2}{2}\left(\left(c_1^2+\frac{1}{s_2^2}\right)+\left(s_1^2+\frac{1}{c_2^2}\right)\xi + \left[\left(\frac{c_2^2}{s_2^2}-s_1^2\right)+\left(\frac{s_2^2}{c_2^2}-c_1^2\right)\xi\right]\sqrt{1+\tan^2 2\theta_F}\right) \end{aligned} \quad (44)$$

For the two bimaximal mixing cases considered above, these formulas are simplified to be

$$\begin{aligned} \tan 2\theta_F &= \frac{2\sqrt{2}(1-\xi)}{1+\xi} \\ m^2_{F_1} &= \frac{3m_F^2}{2}\left(\frac{5}{6}-\frac{1}{6}\sqrt{1+\tan^2 2\theta_F}\right)(1+\xi) \\ m^2_{F_2} &= \frac{3m_F^2}{2}(1+\xi) \\ m^2_{F_3} &= \frac{3m_F^2}{2}\left(\frac{5}{6}+\frac{1}{6}\sqrt{1+\tan^2 2\theta_F}\right)(1+\xi). \end{aligned} \quad (45)$$

for the bimaximal mixing pattern, i.e., $s_1 = s_2 = 1/\sqrt{2}$, and

$$\begin{aligned}\tan 2\theta_F &= \frac{2(1-2\xi)}{3\xi} \\ m_{F_1}^2 &= m_F^2\left(1+\frac{7}{4}\xi - \frac{3}{4}\xi\sqrt{1+\tan^2 2\theta_F}\right) \\ m_{F_2}^2 &= m_F^2\left(1+\frac{5}{2}\xi\right) \\ m_{F_3}^2 &= m_F^2\left(1+\frac{7}{4}\xi + \frac{3}{4}\xi\sqrt{1+\tan^2 2\theta_F}\right)\end{aligned} \qquad (46)$$

for the democratic mixing pattern, i.e., $s_1 = 1/\sqrt{2}$ and $s_2 = \sqrt{2/3}$. In general, the mixing angle θ_F is nonzero, and masses of the three gauge bosons are split after spontaneous symmetry breaking. It is noted that for the bimaximal mixing with $\xi = 1$ and for the democratic mixing with $\xi = 1/2$, the mixing angle θ_F will vanish, and masses of the two gauge bosons $F_\mu^2 = A_\mu^2$ and $F_\mu^3 = A_\mu^3$ will become degenerate.

In the physical mass basis of the leptons and gauge bosons, the gauge interactions of the leptons are given by the following form:

$$\mathcal{L}_F = \frac{1}{2}g_3' F_\mu^{\mathrm{i}}\left(\bar{\nu}_{\mathrm{L}} t^j O_F^{ji}\gamma^\mu \nu_{\mathrm{L}} + \bar{e}_{\mathrm{L}} V_e^{\mathrm{i}}\gamma^\mu e_{\mathrm{L}} - \bar{e}_{\mathrm{R}} V_e^{\mathrm{i}*}\gamma^\mu e_{\mathrm{R}}\right) \qquad (47)$$

with $V_e^{\mathrm{i}} = K_e^j O_F^{ji}$. To be explicit, we have

$$K_e^1 = \begin{pmatrix} 2c_1s_1 & \mathrm{i}c_2(s_1^2-c_1^2) & \mathrm{i}s_2(s_1^2-c_1^2) \\ -\mathrm{i}c_2(s_1^2-c_1^2) & 2c_1s_1c_2^2 & 2c_1s_1c_2s_2 \\ -\mathrm{i}s_2(s_1^2-c_1^2) & 2c_1s_1c_2s_2 & 2c_1s_1s_2^2 \end{pmatrix}, \qquad (48)$$

$$K_e^2 = \begin{pmatrix} 0 & c_1s_2 & -c_1c_2 \\ c_1s_2 & 0 & \mathrm{i}s_1 \\ -c_1c_2 & -\mathrm{i}s_1 & 0 \end{pmatrix}, \qquad (49)$$

and

$$K_e^3 = \begin{pmatrix} 0 & \mathrm{i}s_1s_2 & -\mathrm{i}s_1c_2 \\ -\mathrm{i}s_1s_2 & 2c_1c_2s_2 & (s_2^2-c_2^2)c_1 \\ \mathrm{i}s_1c_2 & (s_2^2-c_2^2)c_1 & -2c_1c_2s_2 \end{pmatrix}, \qquad (50)$$

and

$$\begin{aligned} V_e^1 &= \cos\theta_F K_e^1 + \sin\theta_F K_e^3, \\ V_e^2 &= K_e^2, \\ V_e^3 &= -\sin\theta_F K_e^1 + \cos\theta_F K_e^3 \end{aligned} \qquad (51)$$

As the mixing matrix U_e' does not significantly deviate from the unit matrix, the main features in [6] do not change significantly. In particular, we will obtain, from the current data on the lepton flavor-violating process $\mu \to 3e$ with $\mathrm{Br}(\mu \to 3e) < 1 \times 10^{-12}$ [25], a similar constraint on the SO(3) symmetry-breaking scale,

$$\sigma_{12} > 10^3 v \frac{m_F\sqrt{m_{F_3}^2 - m_{F_1}^2}}{m_{F_1}m_{F_3}} s_1\sqrt{c_1s_2}, \qquad (52)$$

with $v = 246$ GeV the weak symmetry-breaking scale. Specifically, we have

$$\sigma_{12} > 10^3 \frac{v}{2\sqrt{3}}\left(\frac{\tan 2\theta_F(1+\frac{1}{2\sqrt{2}}\tan 2\theta_F)}{1-\frac{1}{24}\tan^2 2\theta_F}\right)^{1/2} \qquad (53)$$

for the bimaximal mixing case, and

$$\sigma_{12} > 10^3 \frac{v}{3}\left(\frac{\tan 2\theta_F(1+\frac{3}{4}\tan 2\theta_F)}{\sqrt{3}(1+\frac{5}{6}\tan 2\theta_F + \frac{1}{8}\tan^2 2\theta_F}\right)^{1/2} \qquad (54)$$

for the democratic mixing case. When the mixing angle θ_F is at the same order of magnitude as the weak mixing angle θ_W, by setting $\tan 2\theta_F \simeq 3/2$, we then obtain

$$\sigma_1 \simeq \sigma_2 \simeq \sigma_3/\sqrt{2} > 0.33 \times 10^3\ v \simeq 81\mathrm{TeV} \qquad (55)$$

for the bimaximal mixing case, and

$$\sigma_1 \simeq \sigma_2 \simeq \sigma_3 > 0.2 \times 10^3\ v \simeq 49\mathrm{TeV} \qquad (56)$$

for the democratic mixing case. Suppose that the SO(3) gauge coupling constant g_3' is at the same order of magnitude as the electroweak coupling constant g; then masses of the three SO(3) gauge bosons are bounded for $\theta_F \sim \theta_W$ to be

$$m_{F_1} > 38\ \mathrm{TeV}, \qquad m_{F_2} > 53\ \mathrm{TeV}, \qquad m_{F_3} > 57\ \mathrm{TeV} \qquad (57)$$

for the bimaximal mixing case, and

$$m_{F_1} > 24\ \mathrm{TeV}, \qquad m_{F_2} > 29\ \mathrm{TeV}, \qquad m_{F_3} > 32\ \mathrm{TeV} \qquad (58)$$

for the democratic mixing case. When the mixing angle becomes very small $\theta_F << 1$, the constraint on the SO(3) symmetry-breaking scale is approximately given by

$$\sigma_1 \simeq \sigma_2 \sim \sigma_3 > 45\sqrt{\tan 2\theta_F}\ \mathrm{TeV}. \qquad (59)$$

Once the mixing angle θ_F is extremely small, at the order of magnitude $\sin\theta_F \sim 10^{-4}$, the SO(3) symmetry-breaking scale can be below 1 TeV, and the SO(3) gauge boson masses may reach to the order of magnitude 300 GeV.

In summary, we have investigated a realistic scheme for lepton masses and mixings within the framework of the gauged SO(3) lepton flavor symmetry discussed recently in [6]. A nearly bimaximal neutrino mixing pattern with maximal CP-violating phase has been derived to explain the solar and atmospheric neutrino data reported recently by the Superkamiokande Collaboration experiment in which the LSND Collaboration results [26] are not considered. This is because if the LSND results are included, it is likely that a sterile neutrino needs to be introduced. [27]. We has also shown that because of the intriguing feature of the vacuum structure of spontaneous SO(3) gauge symmetry breaking, the three Majorana neutrino masses in the scheme are allowed to be nearly degenerate and are large enough for a hot dark matter candidate. Though neutrinoless double beta decay may become unobservably small, the scheme still allows rich interesting phenomena on lepton flavor violations via the SO(3) gauge interactions.

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